Math 105-Technical Mathematics I

Basic Arithmetic (1)

1.1. Arithmetic Operations

I. Fundamental Laws

A. **Cumulative Laws:** \( x + y = y + x; \quad x \times y = y \times x \)

B. **Associative Laws:** \( (x + y) + z = x + (y + z); \quad (x \times y) \times z = x \times (y \times z) \)

C. **Distributive Laws:** \( x \times (y + z) = (x \times y) + (x \times z) \)

II. Order of Operations (from left to right)

A. Operations in **parentheses,** **exponents**

B. **Multiplication** or **division**

C. **Addition** or **subtraction**

* **Please Excuse My Dear Aunt Sally**

* **Examples:**

1. \( (5^2 + 25) \times \frac{(13 - 4)}{3} \)

\[ = (25 + 25) \times \frac{9}{3} = 50 \times 3 = 150 \]

2. \( (5^2 + 25) - \left(\frac{14 - 3}{3}\right) \times 2 \)

\[ = (25 + 25) - \frac{9}{3} \times 2 = 50 - 3 \times 2 = 50 - 6 = 44 \]

* **Try:** 1. \( 40 + (35 - 5^2) \times \frac{55}{30 - 25} = \)
2. \[10 \times 2 + \frac{30 + 40}{7} = \]

3. \[\frac{50 - 25}{5} \times (19 - 11) = \]

4. \[(20 + 7) - \frac{(13 + 3)}{2} = \]

5. \[(1000 - 500) + \frac{2000 - 400}{200} = \]

6. \[(100 + 25) \div (50 - 25) = \]

7. Jack earns $640 for a 40-hour week and Joe earns $570 for a 30-hour week. How much does each earn per hour?

8. In an electric current, the current \(I\) in amps (A) is given by:
\[
I = \frac{200}{6 \times 30 + 20} = \]
calculate the value of the current

9. In an electric circuit, the voltage \(V\) in volts (V), is given by:
\[
V = 4 \times \left(\frac{5 \times 20}{5 + 20}\right) = \]
calculate the value of the voltage

10. In Visual Basic and other computer languages, the arithmetic operations are represented by these symbols:

Addition +; Subtraction -; Multiplication *; Division /

Calculate the following written in computer language:
\[4 + 3 \times 8 \div 12 - 1 = \]
1.2 Multiplying and Dividing Fractions

I. Equivalent Fractions

Two fractions are equivalent if they can each be reduced to the same fraction in lowest terms. Example: \( \frac{75}{100} = \frac{75/25}{100/25} = \frac{3}{4} \)

In this case, the common divisor, called factor, is 25.

II. Prime Number

A number greater than one that is only divisible by one and itself (its only factors are one and itself). Example: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, etc.

III. Divisibility Rules

A. A number is **divisible by two** if the last digit is an even number:

   Example: 0, 2, 3, or 6.

B. A number is **divisible by three** if the sum of the digits is divisible by 3.

   Example: 6, 99, 123, 102, 84.

C. A number is **divisible by five** if the last digit is 0 or 5.

   Example: 25, 30, 90, 125, 240.

D. A number is **divisible by ten** if the last digit is 0.

   Example: 100, 50, 90, 120, 240.

IV. Multiplying Fractions

Simplify the multiplication by factoring out any common factor from the numerator (top) and the denominator (bottom).

Example: \( \frac{3}{15} \times \frac{5}{9} = \frac{1 \times 1}{3 \times 3} = \frac{1}{9} \)
V. Dividing Fractions

Dividing by a fraction would yield the same result as multiplying by the reciprocal of that fraction.

Example:

\[
5 \div \frac{5}{15} = 5 \times \frac{15}{5} = 5 \times 3 = 15
\]

\[
6 \div 10 = 6 \times \frac{1}{10} = \frac{6}{10} = \frac{3}{5}
\]

\[
9 \div \frac{1}{5} = 9 \times \frac{5}{1} = 45
\]

\[
121 \div \frac{11}{12} = 121 \times \frac{12}{11} = 11 \times 12 = 132
\]
9. In an electric circuit, the power $P$ in watts (W) is given by:

$$P = \frac{1}{100} \times \frac{1}{100} 400 =$$

Calculate the value of power $P$.

10. In an electric circuit, the voltage $V$ in volts (V) is given by:

$$V = \frac{3}{5} \times 70 =$$

Calculate the value of the voltage.
1.3 Adding Fractions

In operation of addition or subtraction involving fractions, it is necessary to first determine the common denominator before adding or subtracting the numerators. Ideally, the best common denominator to use is the least common multiple (such that all factors from each and every denominators involved are considered/taken care of). Once the common denominator (least common multiple) is determined, then multiply the top(numberator) and bottom(denominator) of each fraction by what is the missing factor. Once all fraction has the common denominator, then proceed to adding all numerators and keep the same denominator.

Example: \[\frac{3}{4} + \frac{5}{6} = \frac{3 \times 3}{4 \times 3} + \frac{5 \times 2}{6 \times 2} = \frac{9}{12} + \frac{10}{12} = \frac{9 + 10}{12} = \frac{19}{12}\]

*\frac{19}{12}\] is an improper fraction (if numerator is greater than denominator)

\[\frac{19}{12}\] is equivalent to \(1\frac{7}{12}\)

In this case, \(4 = 2 \cdot 2\) \(6 = 2 \cdot 3\)

To insure that all factors are taken care of, we need to multiply \(2 \cdot 2 \cdot 3 = 12\)

\(\therefore 12\) is the least common multiple or the best common denominator to use.

Try: 1. \(\frac{1}{6} + \frac{5}{8} = \)

2. \(\frac{2}{3} + \frac{1}{6} = \)

3. \(\frac{11}{20} + \frac{3}{16} = \)

4. \(\frac{3}{20} + \frac{1}{6} - \frac{1}{15} = \)

5. \(\frac{2}{5} + \frac{3}{4} = \)

6. \(\frac{6}{1000} + \frac{25}{100} + \frac{6}{500} = \)

7. \(1 - \frac{23}{50} + \frac{13}{100} = \)

8. \(3 - \frac{6}{15} + \frac{3}{20} = \)

9. \(\frac{1}{2} + \frac{2}{9} + \frac{2}{3} = \)

10. \(\frac{7}{50} + \frac{6}{25} \times \frac{5}{12} = \)
11. \( \frac{1}{5} + \frac{1}{4} \div 3 = \)

12. \( \frac{16}{25} - \frac{3}{10} \div 6 = \)

13. \( (1 + \frac{4}{7}) \div (1 - \frac{4}{7}) = \)

14. \( \frac{1}{2} + \frac{1}{4} \times \left( \frac{4}{5} - \frac{1}{3} \right) = \)

15. \( \frac{6}{10} - \frac{1}{20} + \frac{9}{15} \div 3 = \)

16. \( \frac{3}{200} + \frac{3}{50} - \frac{1}{3} \times \frac{9}{40} = \)

17. In a circuit of parallel resistors, \( \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \)

\( R \), resistance, is in unit of ohms (Ω)

Calculate \( R \), resistance, in ohms (Ω): given \[ \frac{1}{R} = \frac{1}{20} + \frac{1}{50} + \frac{1}{30} \]

Calculate \( R \), resistance, in ohms (Ω): given \[ \frac{1}{R} = \frac{1}{5} - \frac{1}{10} - \frac{1}{15} \]

Calculate \( R \), resistance, in ohms (Ω): given \[ R = 10 + \frac{10 \times 5}{10 + 5} \]

18. Calculate the following expression in computing language:

a. \( 1 \div 10 \times 5 \div (3 / 2 - 1) \times 20 \div 3 \)

b. \( (4 \div 5 - 1/10) \div (7/10 + 1/5) \times 45 \div 100 \)